Your calculator as a weapon

Daniel V Mathews

Daniel.Mathews@monash.edu



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newest threat?

June 8, 2015	
Liam Tung	



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Dangerous minds: Are maths teachers Australia's newest threat?

June	8,	201	5	
Liam	i T	ung		

Comments 96 Comments



Arts + Culture Business + Economy Education Environment + Energy Health + Medicine Politics + Society Scient

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Paranoid defence controls could criminalise teaching encryption

May 19, 2015 2.37pm AEST



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Australia's Act of Intellectual Terrorism: DTCA 2012

- Kevin B Korb

In October 2012 the Australian parliament passed the Defence Trade Controls Act. The stated purposes of the act are unobjectionable: implementing the prior Australa-United States Defense Trade Cooperation Treary, simplifying defence-related trade between Australia, the US and the UK, and diptening the regulation of intanpible transfers of military goods, reflecting the growth of the internet in communications. Unfortunately, these good intentions have ted the Australian government to adopt an extraordinarily broad definition of military goods and to impose an impossibly harsh regulatory regime on activities concerning them, to the port that what is today ordinary academic research into, for example, Bayesian



Response to Australia's Defence Trade Controls Act

July 6, 2015

We are deeply concerned about Australia's Defence Trade Controls Act (DTCA). The act prohibits the "intangible supply" of encryption technologies, and hence subjects many ordinary teaching and research activities to unclear, potentially severe, export controls. As an international organization of cryptographic researchers and educators, we are concerned that the DTCA criminalizes the very essence of our association: to advance the theory and practice of cryptography in the service of public welfare.

We affirm that the public welfare of Australians - and society in general - is best served by open research and education in cryptography and cybersecurity. Open, international scientific collaboration is responsible for the encryption technologies that are now vital to individuals, businesses, and world governments alike. The current legislation cuts off Australia from the international cryptographic research community and jeopardizes the supply of qualified workforce in Australia's growing cybersecurity sector.

We call on Australia to amend their export control laws to include clear exemptions for scientific research and for education

IACR Member Signatories (219): add your signature!

- · Christian Cachin. President of the IACR. IBM Research Zurich. Switzerland
- Nigel Smart, Vice President IACR, University of Bristol, United Kingdom

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Academics like Kevin Korb are nervous that "overly broad" definitions in the DSGL could land them in court for teaching cryptography... and a number of other fields.



I'll try to:

- Show you some of these laws and try to make some sense of them
- Explain some related ideas from mathematics and cryptography
- Present a facetious-but-not-that-facetious argument that your calculator could be regarded as a dual-use military-civilian item ("weapon")

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Raise some broader issues

Warning/disclaimer:

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- 96 pages long
- Main provisions were due to come into effect 16 May 2015



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Most controversial provision: section 10

Offence — supply of DSGL technology



Dealings in items in the Defence and Strategic Goods List Part 2 Primary offences Part 2 Dealings in items in the Defence and Strategic Goods List Division 1 Primary offences

Section 10

Section 10

to supply DSGL technology. There is a process for becoming a registered broker.

Division 1-Primary offences

10 Offence-supply of DSGL technology

- (1) A person (the supplier) commits an offence if:
 - (a) the supplier supplies DSGL technology to another person; and
 - (b) either:
 - (i) the supply is from a place in Australia to a place outside Australia; or
 - (ii) if the supply is the provision of access to DSGL technology — at the time of the provision of access, the supplier is in Australia and the other person is outside Australia; and
 - (c) either:
 - (i) the supplier does not hold a permit under section 11 authorising the supply of the DSGL technology, or
 - (ii) the supply of the DSGL technology contravenes a condition of a permit that the supplier holds under section 11; and
 - (d) there is no notice in force under subsection 14(1) in relation to the supplier and the supply.
 - Penalty: Imprisonment for 10 years or 2,500 penalty units, or both.

Exceptions

- (1A) Subsection (1) does not apply if:
 - (a) the supply is not the provision of access to DSGL technology; and
 - (b) the supply is made orally; and

- (c) the supply is neither for a military end-use nor for use in a Weapons of Mass Destruction program.
- Note: A defendant bears an evidential burden in relation to the matter in subsection (1A): see subsection 13.3(3) of the *Criminal Code*.
- (2) Subsection (1) does not apply if:
 - (a) the supply is of DSGL technology in relation to original goods; and
 - (b) the supply is by an Australian Community member or by a member of the United States Community; and
 - (c) the supply is to an Australian Community member or a member of the United States Community; and
 - (d) the supply is for an activity referred to in Article 3(1)(a), (b), (c) or (d) of the Defense Trade Cooperation Treaty; and
 - (e) at the time of the supply, the original goods are listed in Part 1 of the Defense Trade Cooperation Munitions List; and
 - (f) at the time of the supply, the original goods are not listed in Part 2 of the Defense Trade Cooperation Munitions List.
 - Note: A defendant bears an evidential burden in relation to the matter in subsection (2): see subsection 13.3(3) of the Criminal Code.
- (3) Subsection (1) does not apply if:
 - (a) the DSGL technology is supplied by or to a person who is a member of the Australian Defence Force, an APS employee, an employee of ASIO, an employee of ASIS, a member or special member of the Australian Federal Police or a member of the police force of a State or Territory, and
 - (b) the supply occurs he or she supplies the DSGL technology in the course of his or her duties as such a person.
 - Note: A defendant bears an evidential burden in relation to the matter in subsection (3): see subsection 13.3(3) of the Criminal Code.
- (3A) Subsection (1) does not apply if:
 - (a) the supply is of DSGL technology within the scope of Part 2 of the Defence and Strategic Goods List; and
 (b) the supply is preparatory to the publication of the DSGL
 - (b) the supply is preparatory to the publication of the DSGL technology to the public or to a section of the public; and

Dealings in items in the Defence and Strategic Goods List Part 2 Primary offences Division 1

Section 11

- (c) there is neither a notice in force under subsection 14B(1), nor a notice in force under subsection 14C(1), in relation to the supplier and the DSGL technology.
- Note: A defendant bears an evidential burden in relation to the matter in subsection (3A): see subsection 13.3(3) of the Criwinal Code.
- (4) Subsection (1) does not apply in the circumstances prescribed by the regulations for the purposes of this subsection.
 - Note: A defendant bears an evidential burden in relation to the matter in subsection (4): see subsection 13.3(3) of the Criminal Code.

Geographical jurisdiction

(5) Section 15.2 of the Criminal Code (extended geographical jurisdiction – category B) applies to an offence againstrabation (1).

Definition

(6) In this section:

place includes:

- (a) a vehicle, vessel or aircraft; and
- (b) an area of water; and
- (c) a fixed or floating structure or installation of any kind.

11 Permits for purposes of section 10

 A person may apply to the Minister for a permit under this section to supply DSGL technology to another person.

lote: Section 66 sets out application requirements.

- (2) Without limiting subsection (1), an application by a person under that subsection may do one or more of the following:
 - (a) cover 2 or more supplies by the person;
 - (b) cover one or more supplies by the person for a period described in the application;
 - (c) cover one or more supplies by the person for a project described in the application.

Defence Trade Controls Act 2012 No. 153, 2012 15

A summary of the offence:

- The "supply" of "DSGL technology" overseas without a permit is an offence.
- (But military technology may flow freely US ↔ Australia.)
- Exemptions for "supplies" which are "oral" or "preparatory to publication" to (a section of) the public.
- Maximum penalty: 10 years imprisonment or \$450,000 fine

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- (a) includes supply by way of sale, exchange, gift, lease, hire or hire-purchase; and
- (b) in relation to DSGL technology—includes provide access to DSGL technology.

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Existing Gaps – Intangible Supply

- A permit is required if an Australian physically exports a controlled virus. However, if they email instructions on how to produce or enhance that virus, no permit is currently required.
- The Australian Government has no visibility or control over the electronic export of this information, including whether it is potentially destined for a biological weapons program.

"Supply":

- Need not be for payment.
- Can include *email explanations*.

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"DSGL technology":

• Refers to the Defence and Strategic Goods List (DSGL)

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DSGL is essentially in two parts:

- Part 1: Munitions list
- Part 2: Dual-use list

Part 1A—Preliminary	1
Division 1—Preliminary	1
1 Name	1
2 Authority	1
Division 2—Preface	2
Division 3—Notes	3
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Category 5 — Telecommunications and "information security"

Part 2 — "INFORMATION SECURITY"

5A2 Systems, Equipment and Components

5A002 "Information security" systems, equipment and components therefor, as follows: ...

- a. Systems, equipment, application specific "electronic assemblies", modules and integrated circuits for "information security", as follows and components therefor specially designed for "information security": ...
 - 1. Designed or modified to use "cryptography" employing digital techniques performing any cryptographic function other than authentication, digital signature or the execution of copy-protected "software", and having any of the following: ...
 - a. A "symmetric algorithm" employing a key length in excess of 56 bits; or ...
 - b. An "asymmetric algorithm" where the security of the algorithm is based on any of the following: ...
 - 1. Factorisation of integers in excess of 512 bits (e.g., RSA);
 - Computation of discrete logarithms in a multiplicative group of a finite field of size greater than 512 bits (e.g., Diffie-Hellman over Z/pZ); or

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 - 3. Discrete logarithms in a group other than mentioned in 5A002.a.1.b.2. in excess of 112 bits (e.g., Diffie-Hellman over an elliptic curve); ← □ ▷ ← ⊡ ▷ ← ⊡ ▷ ← ≡ ▷ ↓ ≡ ○ ♀ (♡

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DSGL section 5A002.a.1

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 Any "system" or "equipment" which does sufficiently strong encryption is covered.

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Note there are many exceptions, not all very clear.

DSGL

• DTCA:

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 - US/Australian military, intelligence, police.



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Political/moral/legal/policy questions:

• Is this idea right?

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Applied cryptography question:

• How strong are the specifications in the DSGL? WEAK! Mathematics/computer science questions:

In abstract algebra there are things called groups.



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- The *positive real numbers* with the operation of *multiplication* (ℝ₊, ×).

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 The set of "remainders after dividing by 12" or *integers* modulo 12 forms a group under addition (ℤ₁₂, +).

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$$1 + 1 = 2 11 + 2 = 1 12 = 0 3 + 6 = 9 9 + 8 = 5 -3 = 9$$

 Similarly, we can take "remainders after dividing by n" and obtain *integers modulo n* with addition (ℤ_n, +).

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 (Except you have to remove 0; you can't undo multiplication by zero.)

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- E.g. $p = 7: (\mathbb{Z}_{7}^{*}, \times).$
- $1 \times 2 = 2$
- $\mathbf{3\times 6}=\mathbf{18}=\mathbf{4}$

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A group does not care if its operation is addition, multiplication, or anything else!

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From now on write group operations by \cdot (or juxtaposition).





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Careful!





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Careful!

• In $(\mathbb{Z}_7, +)$, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 10 = 3$.
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- In $(\mathbb{Z}, +)$, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 10$.
- In (\mathbb{R}_+, \times) , $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.

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Whatever operation we have, we can do it repeatedly and obtain *discrete exponentials*.

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Careful!

- In $(\mathbb{Z}_7, +)$, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 10 = 3$.
- In $(\mathbb{Z}_{7}^{*}, \times)$, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 = 4$.

• In
$$(\mathbb{Z}, +)$$
, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 10$.

• In
$$(\mathbb{R}_+, \times)$$
, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.

Whatever operation we have, we can do it repeatedly and obtain *discrete exponentials*.

• In
$$(\mathbb{Z}_7^*, \times)$$
, $2^5 = 4$.

• In
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, $2^5 = 32$.

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• Discrete logarithm in \mathbb{Z} is also known as... division.

discrete logarithms are hard to compute in (\mathbb{Z}_p^*, \times) when p is large.

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discrete logarithms are hard to compute in (\mathbb{Z}_p^*, \times) when p is large.

It's easier to compute 3⁵ mod 7 than it is to compute log₃ 5

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Question

Despite such public conditions, can A and B establish a shared secret?







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 - These calculations are "easy" (exponentiation mod p).
 - Now p, g, g^a, g^b are publicly known but a is known only to A and b is known only to B.

$$p \text{ prime}$$

 $g \in \mathbb{Z}_p^*$
 $(g^b)^a = g^{ab}$ g^a, g^b $B \qquad (g^a)^b = g^{ab}$

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- A calculates $(g^b)^a = g^{ab} \mod p$. B calculates $(g^a)^b = g^{ab} \mod p$.
 - The number $g^{ab} \pmod{p}$ is A and B's shared secret.



Security of the key exchange:

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Security of the key exchange:

An eavesdropper Eve must compute shared secret g^{ab} from knowledge of g^a, g^b, p, g only, in order to listen in.



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- An eavesdropper Eve must compute shared secret g^{ab} from knowledge of g^a, g^b, p, g only, in order to listen in.
- Finding *a* or *b* from g^a, g^b would allow Eve to compute g^{ab}, but this requires finding discrete logarithms modulo *p*.

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$$a = \log_g g^a, b = \log_g g^b$$

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3. Discrete logarithms in a group other than mentioned in 5A002.a.1.b.2 in excess of 112 bits [sic]

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- Has infinitely many elements far more than 2¹¹²!
- Discrete logarithm is just *division*.
 - E.g. log₃ 18 = 6.

"Cryptography"... based on... An "asymmetric algorithm" where the security of the algorithm is based on any of the following:...

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Are there any encryption algorithms based on division?

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Are there any encryption algorithms based on division?

Yes!

(Are there any good ones? Not that I know of.)

Dan's Basic Algorithm using Division (BAD)



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To encrypt a message:

- Onvert the message to a number *m*.
- Choose a secret encryption key k.
- Multiply m by k to obtain the cyphertext c = mk.

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WORST. ALGORITHM. EVER.

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But, the BAD algorithm is:

- A cryptographic algorithm
- Asymmetric (encryption different from decryption)
- Security is based on division, i.e. discrete logarithm in a group with more than 2¹¹² elements.

Hence (if not public domain or "basic scientific research" etc):

DUAL-USE CIVILIAN-MILITARY ITEM.

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But...

- There are serious issues with these laws.
- These laws affect university education and research directly.
- A small part of broader issues re security, transparency, national security, civil liberties.

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- Laws should be written & implemented by people who understand them.
- Technical knowledge is important in debates on these topics.
- Even if badly written laws are unlikely to be used in bad ways, they *could* be so used.
- We shouldn't have badly written laws in the first place!

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How did we get here?

- 1990s "crypto wars" over US encryption policy
- US Export controls (ITAR)
- International arms control: Wassenaar Arrangement
- Australian DSGL
- 2007 Australia-US Defence Trade Cooperation Treaty

Finally...

Don't stop doing mathematics!

THANKS FOR LISTENING.