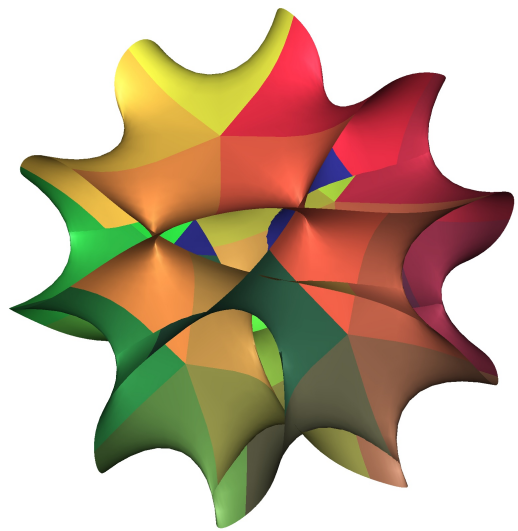
Mathematics and Mathematicians

Daniel V Mathews

Daniel.Mathews@monash.edu



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Also, pure mathematics.

Some important mathematicians

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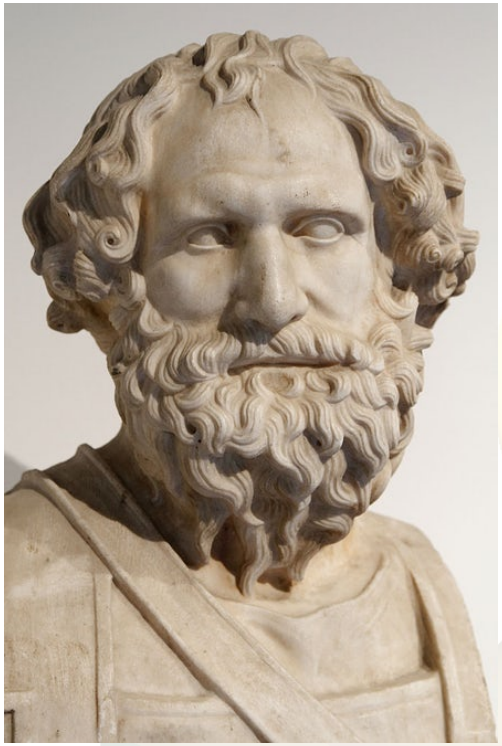

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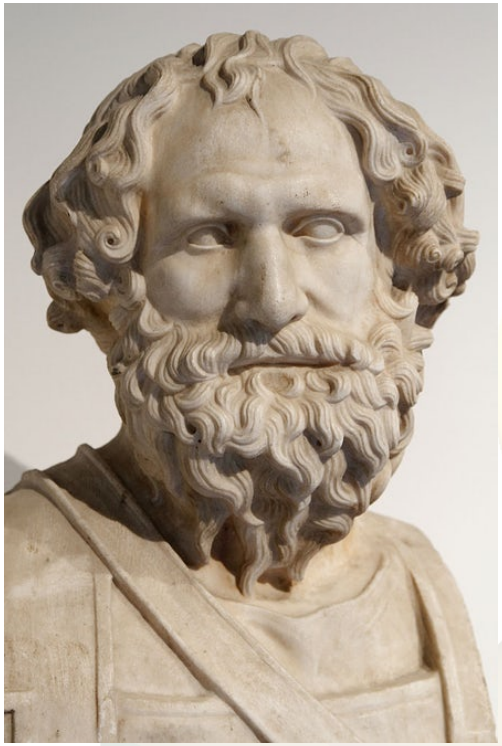

Archimedes

(Greece, 3rd century BCE)

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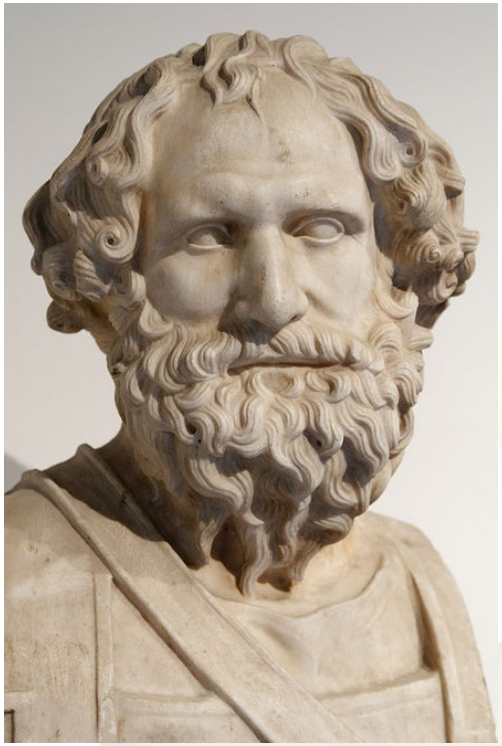

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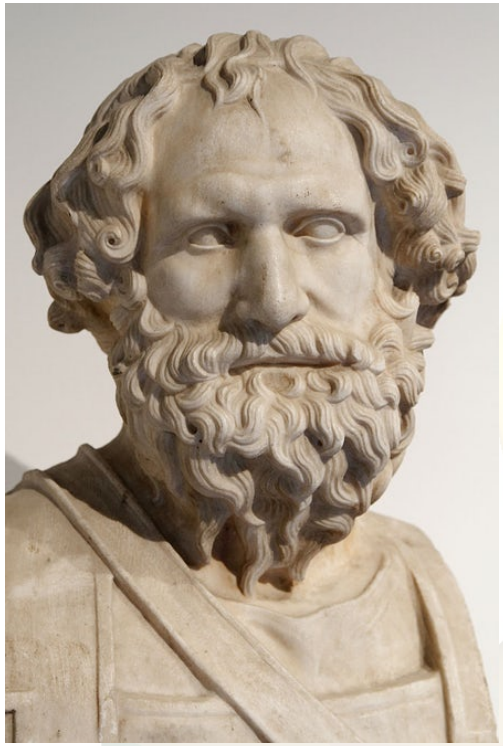

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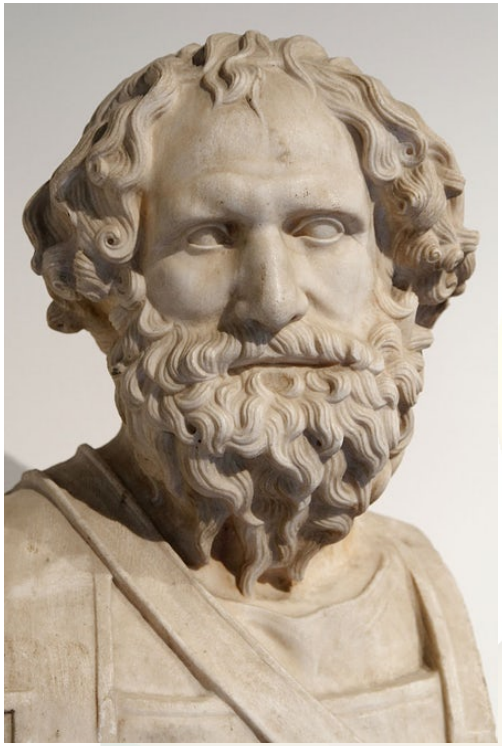

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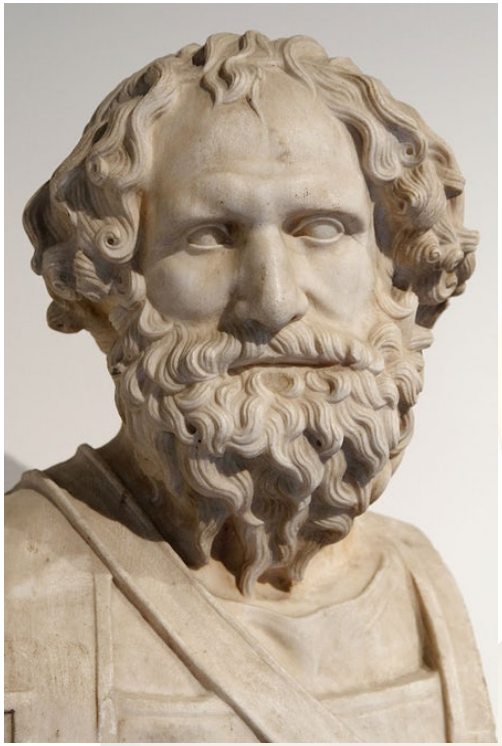

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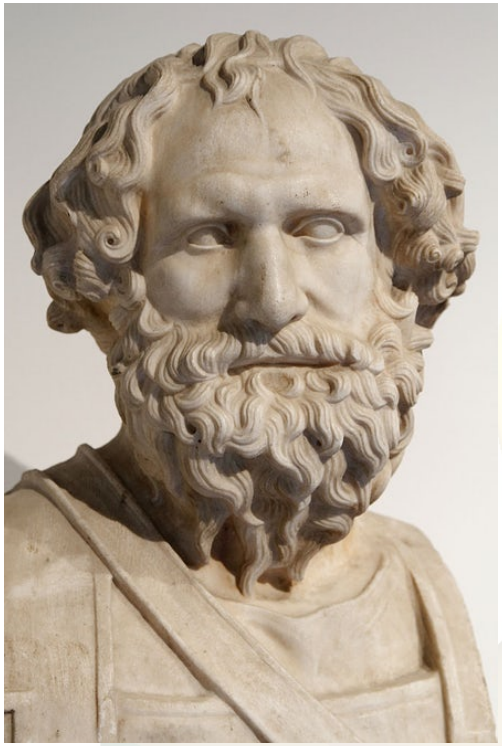

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“Apres cela il se trouvera, j’espere, des gens qui trouvent leur profis a dechiffrer tout ce gachis."

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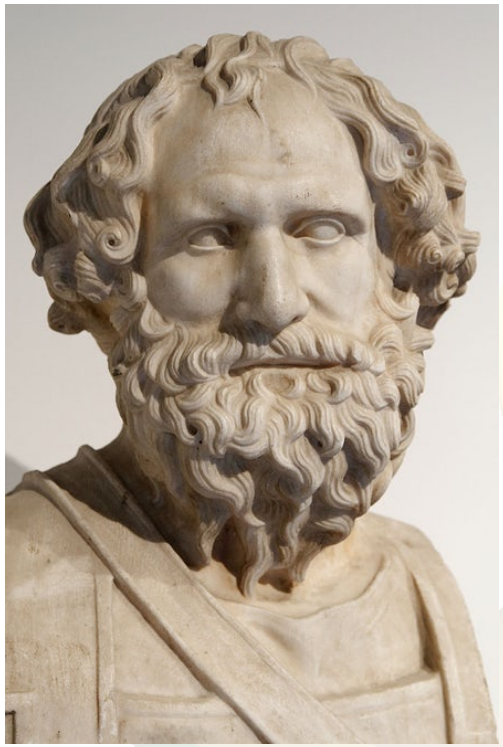

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“After this, I hope there will be people who will profit by deciphering all this mess."

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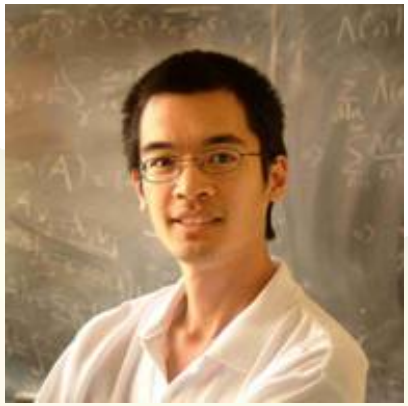

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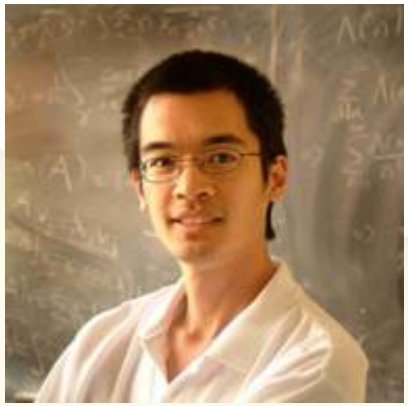

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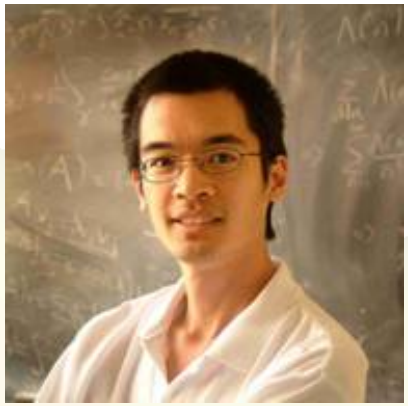

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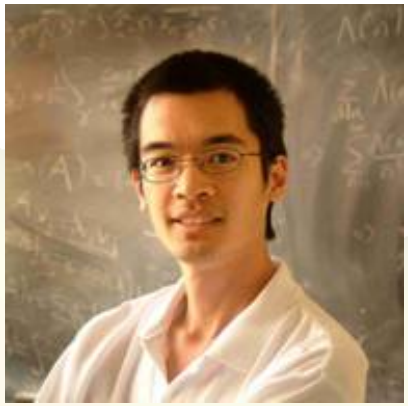

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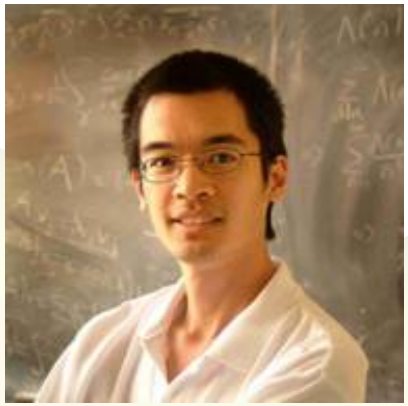

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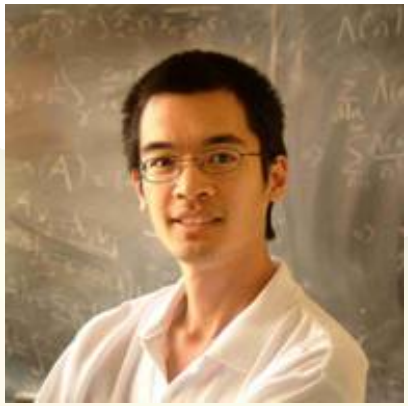

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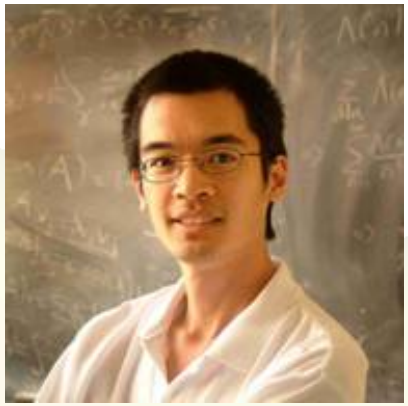

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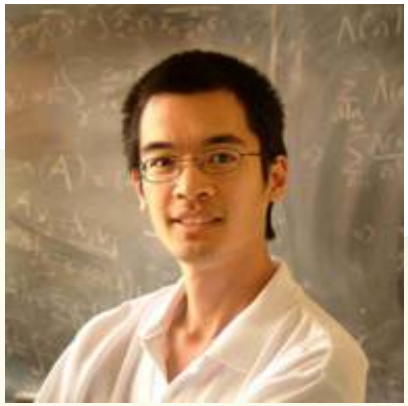

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Maryam Mirzakhani

(Iranian, 1977 – 14 July 2017)

"I did poorly in math for a couple of years in middle school; I was just not interested in thinking about it... The beauty of mathematics only shows itself to more patient followers."

What is mathematics about?

Mathematics is:

- Often applied to the real world
  (but not all of it!)
- Often pure mathematics done for its own sake.

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Mathematics is about:

- Truth
- Deep understanding
- Logic, deduction
- Creativity, intuition
- Elegance, beauty
- Proof
- Understanding the world.

What is topology?

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A type of "geometry" where we forget about all lengths, angles, areas, etc — only study the "shape"' of objects.

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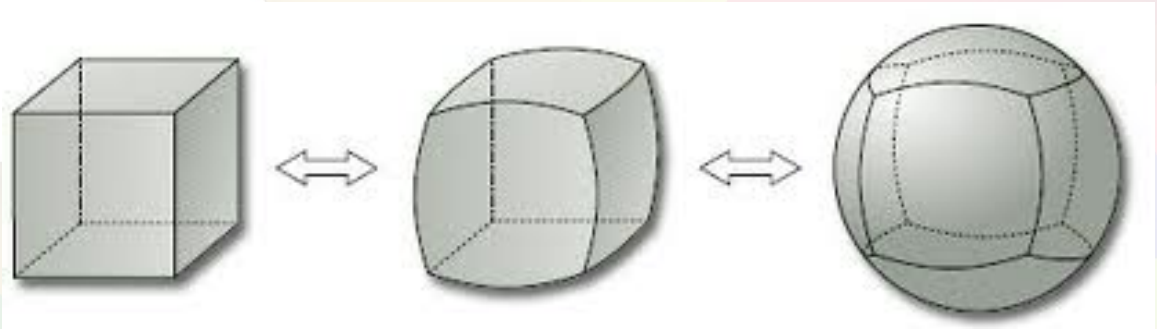


Source: Ágnes Szilárd

A cube is equivalent to a sphere...

What is topology?

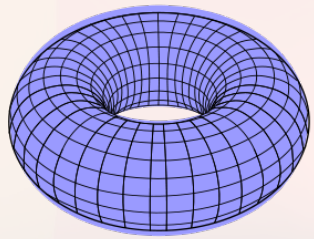
... a heart is equivalent to a disc...

What is topology?

... a heart is equivalent to a disc...

... And a donut is equivalent to a coffee cup.

Source: XKCD



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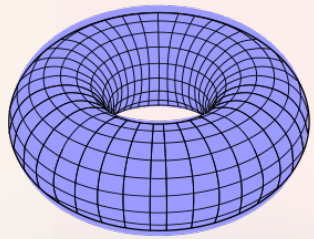

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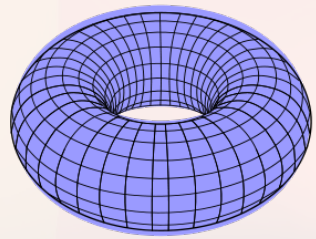

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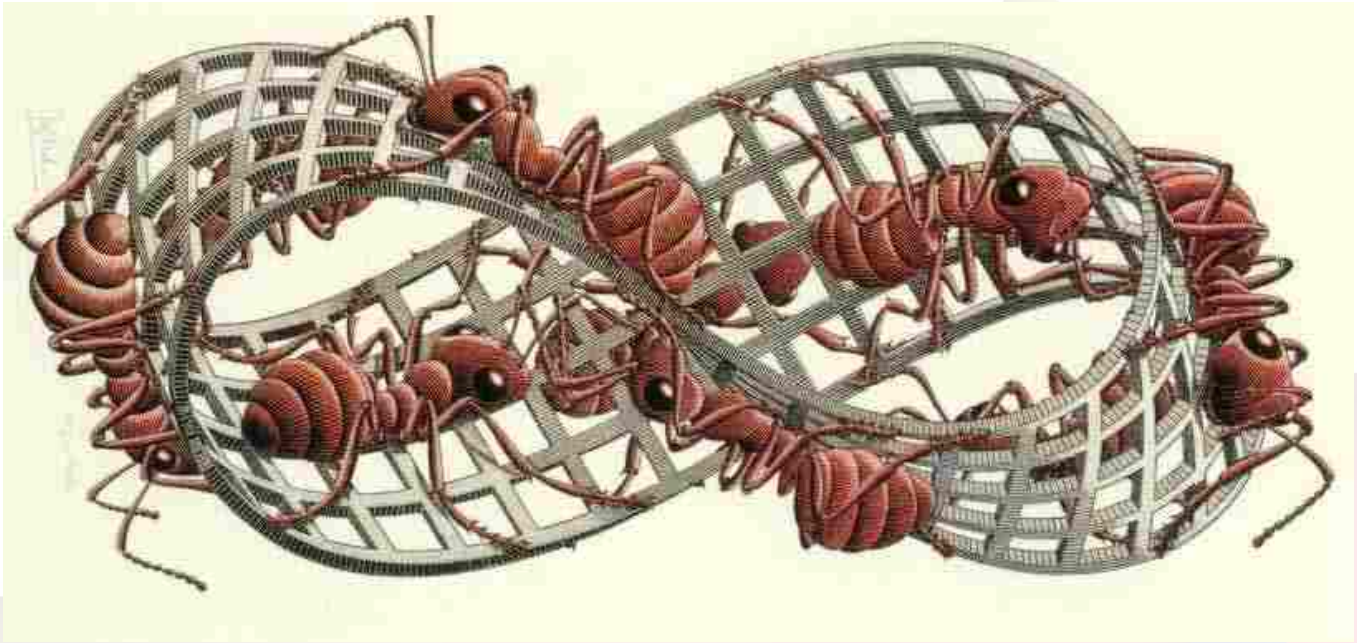
“Rubber sheet geometry."

- New ways of thinking about geometry and space.

Topology in art



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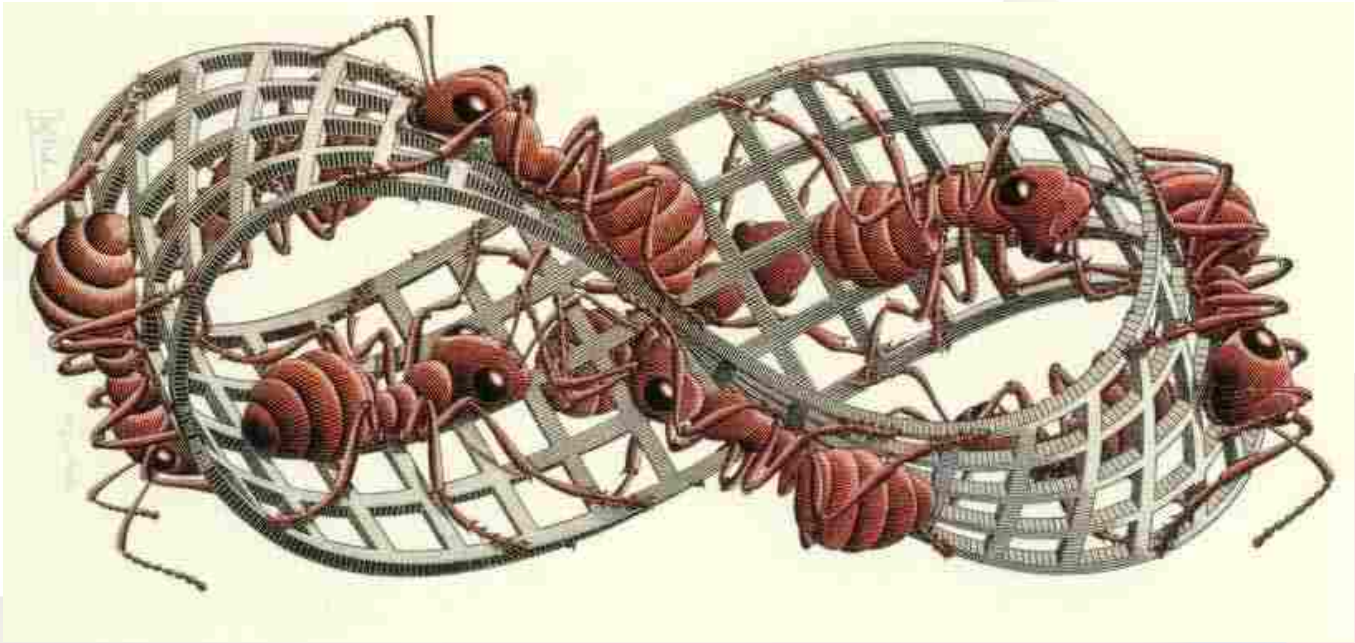
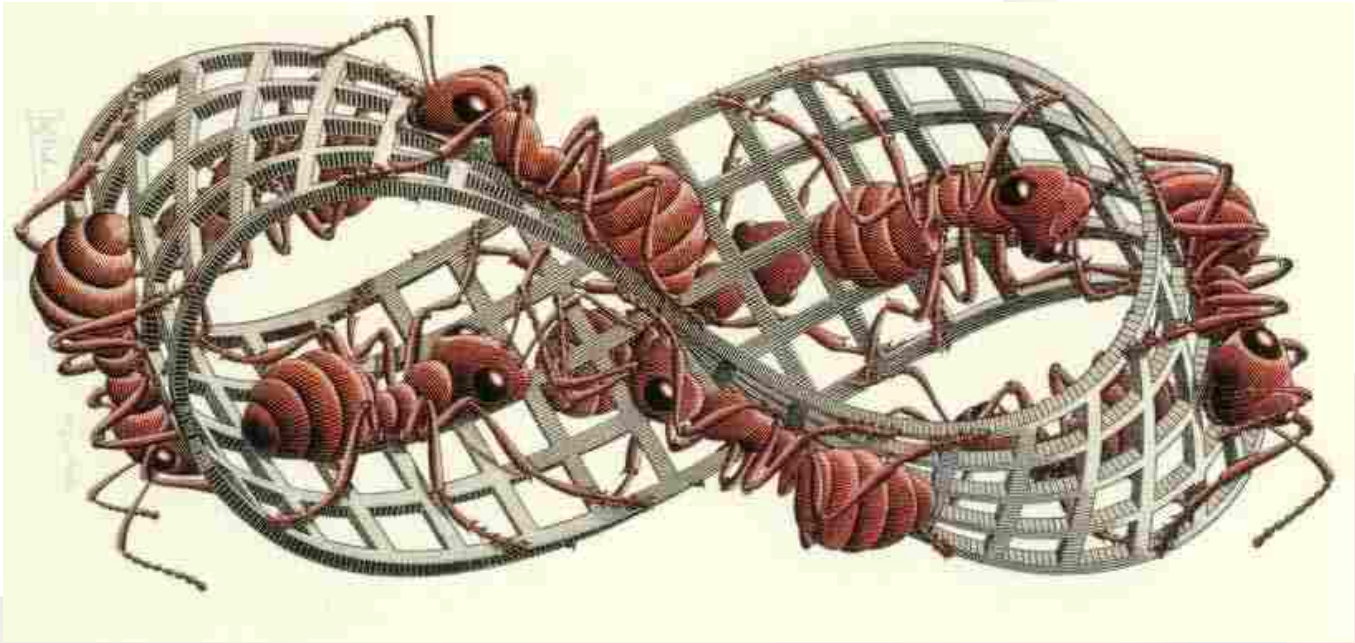


The Möbius strip

Topology in art

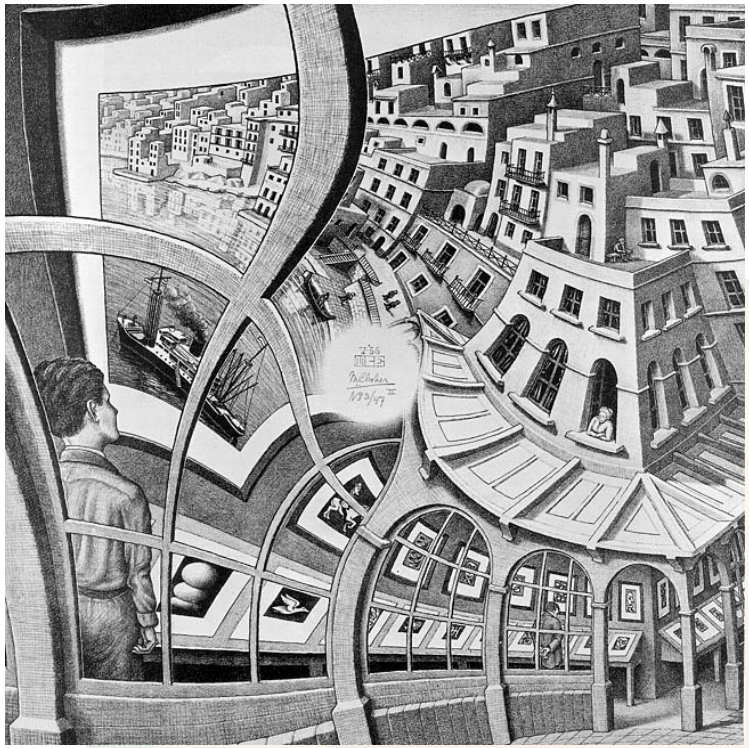


The Möbius strip

- *Non-orientable*

Topology in art

M.C. Escher: *Print gallery*.



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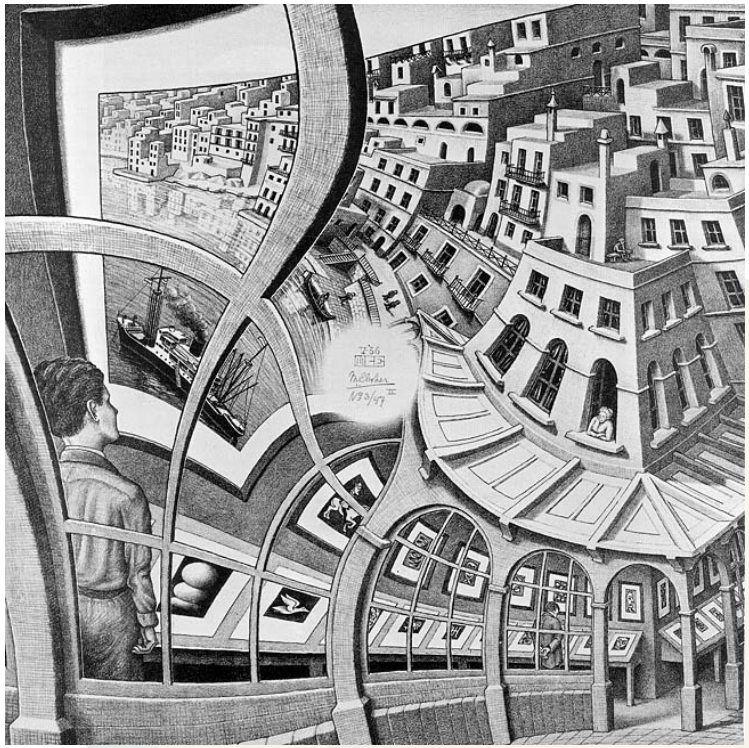

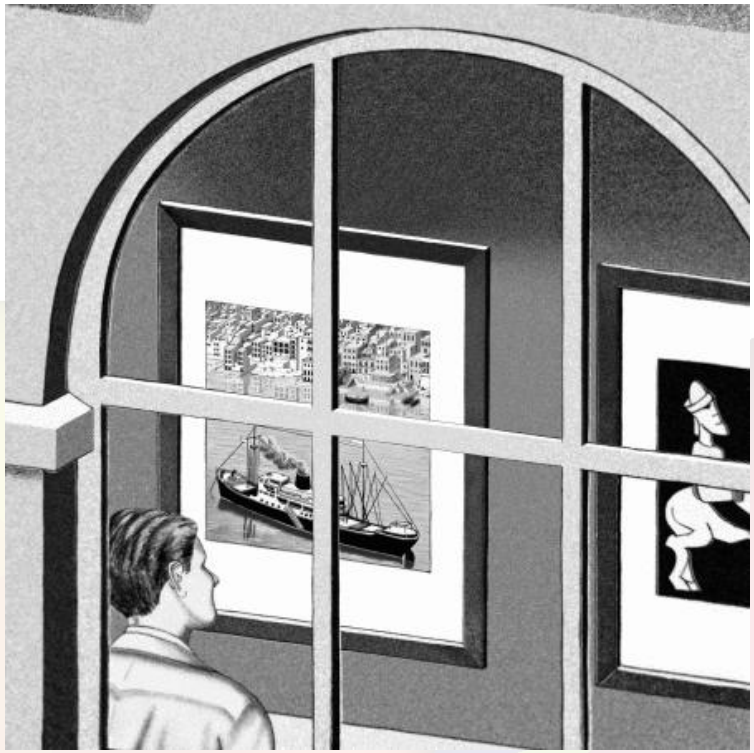

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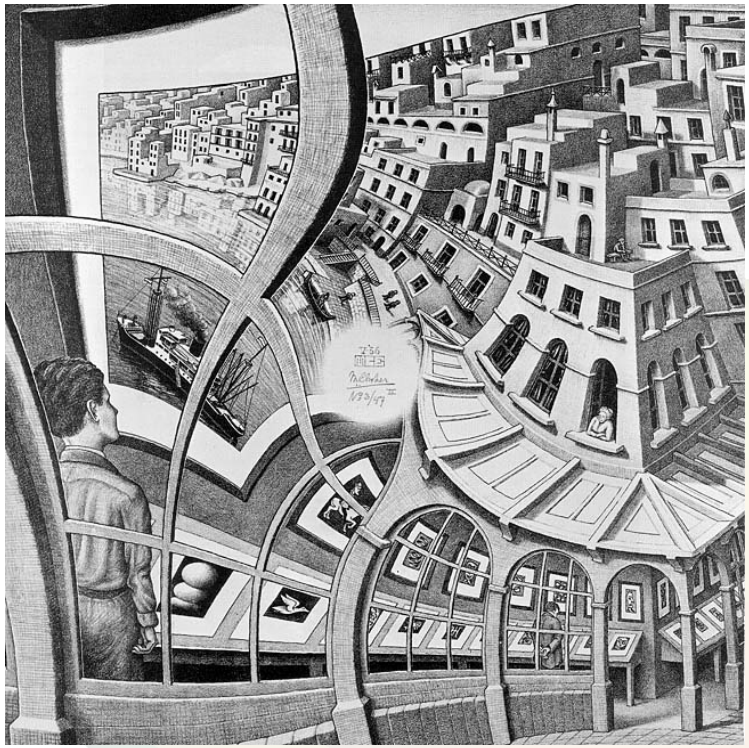

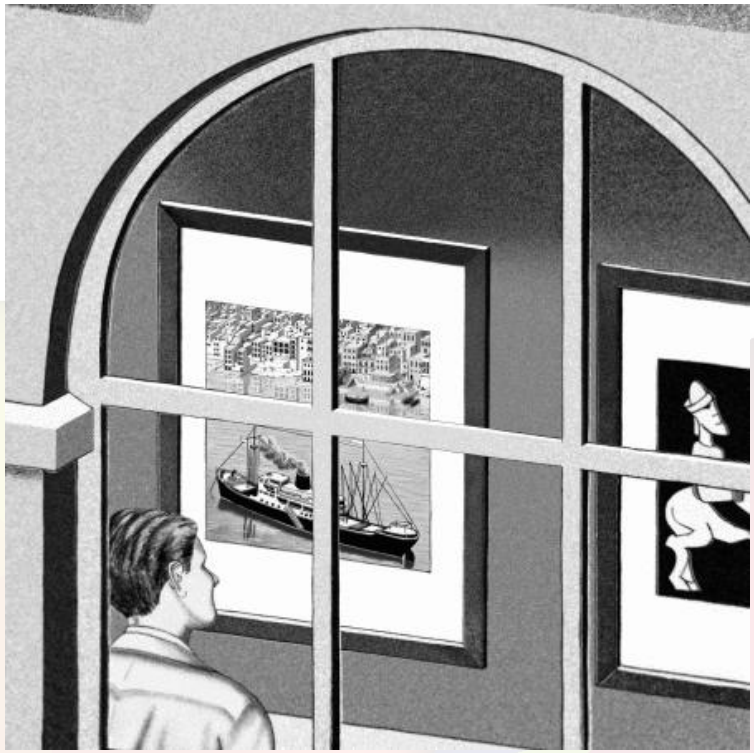

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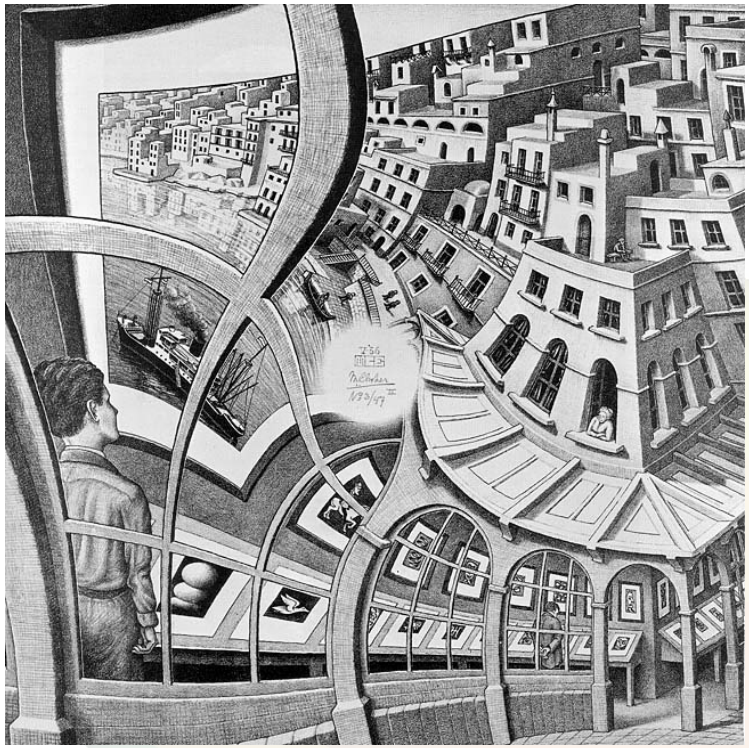

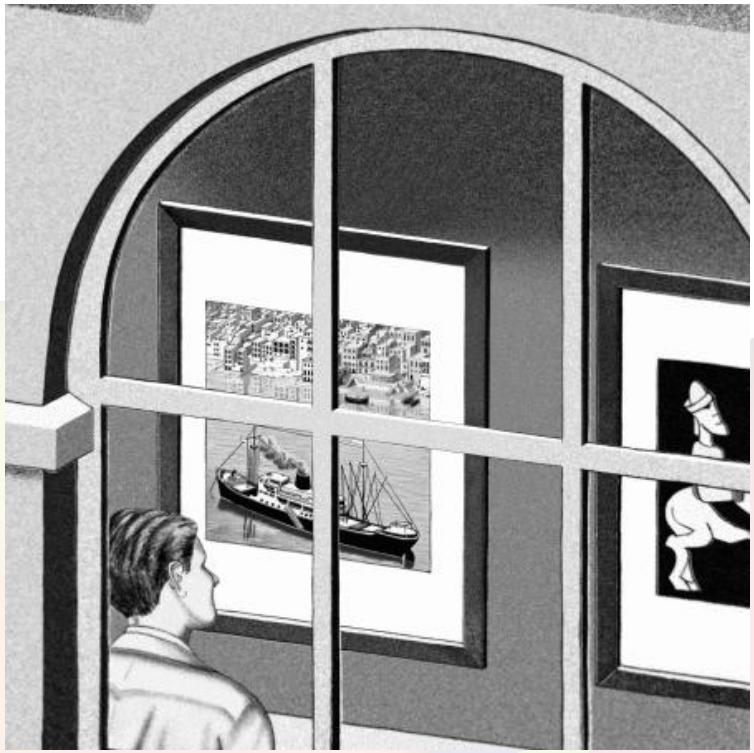

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  - (Mirzakhani was a master of this kind of geometry!)
- What happens if you zoom in?

Topological video games...

Wraparound: a gameplay variation on video games...

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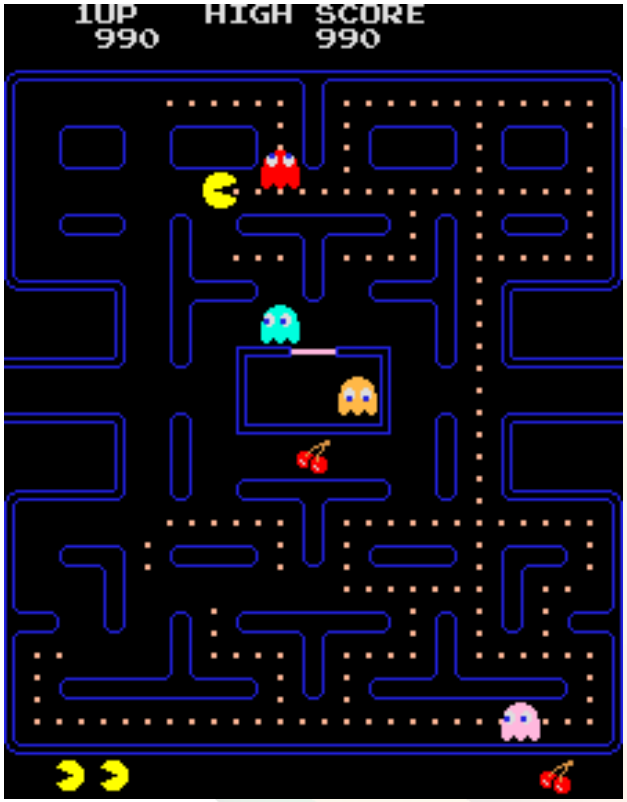

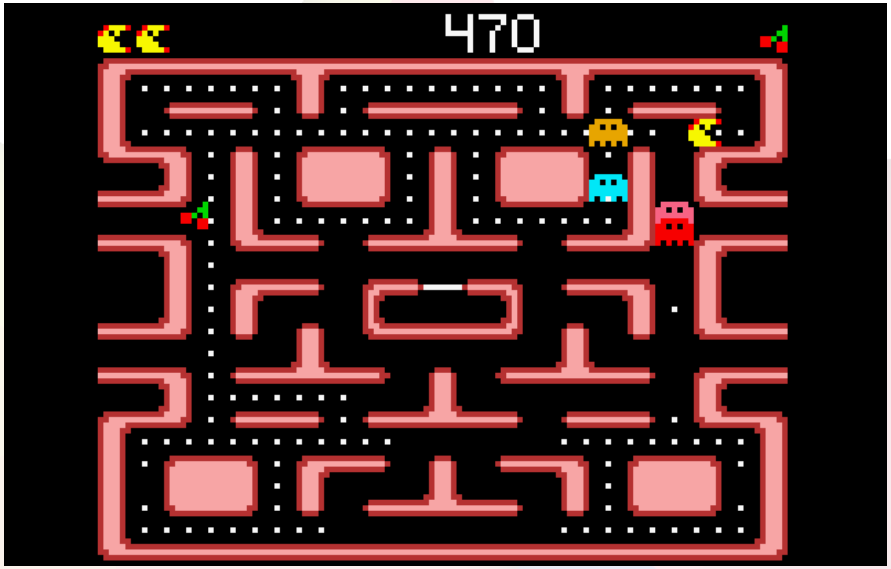

Pac Man & Ms. Pac Man

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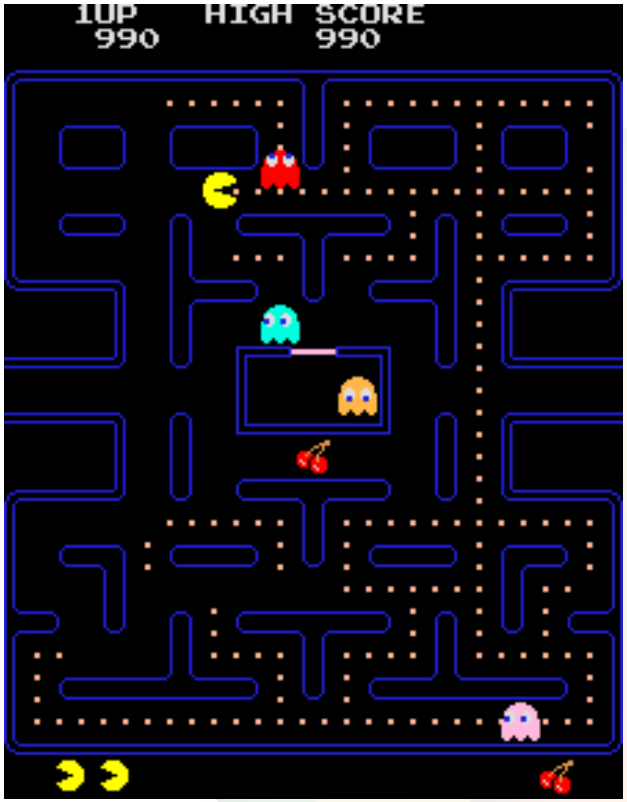

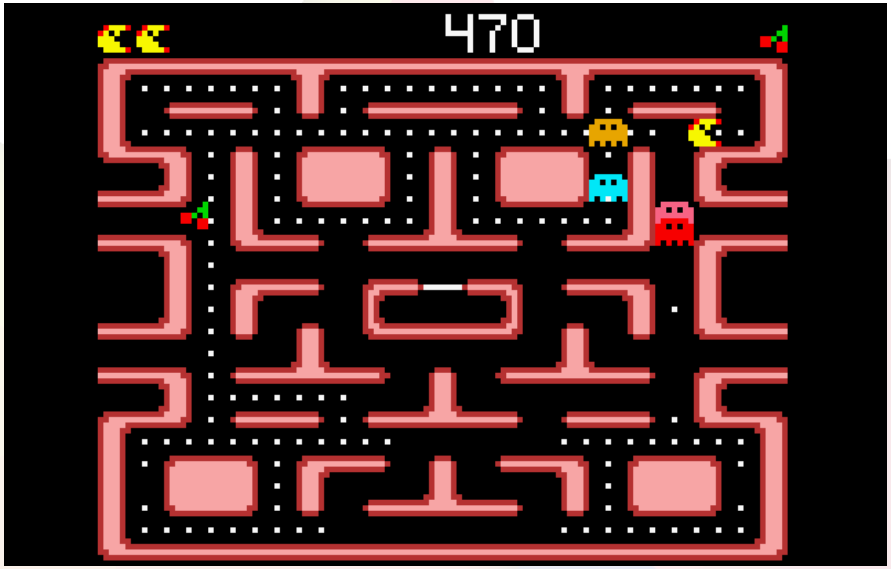

Pac Man & Ms. Pac Man

What is the topology of Pac Man?

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Idealised Pac Man: exit the screen at any point & re-enter opposite...

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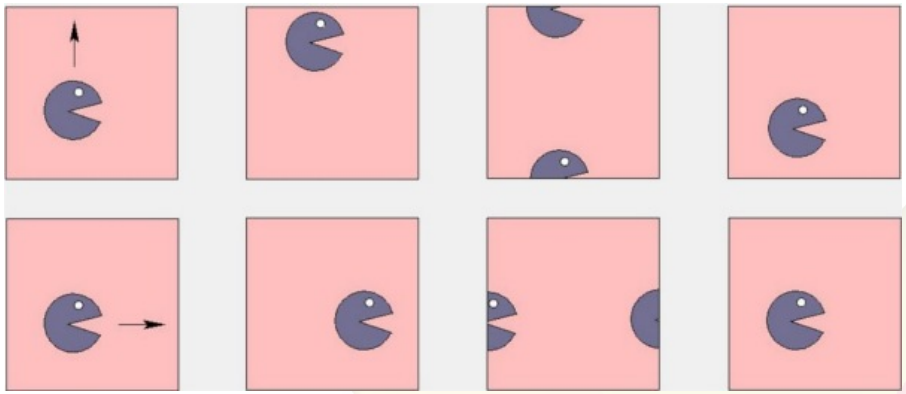

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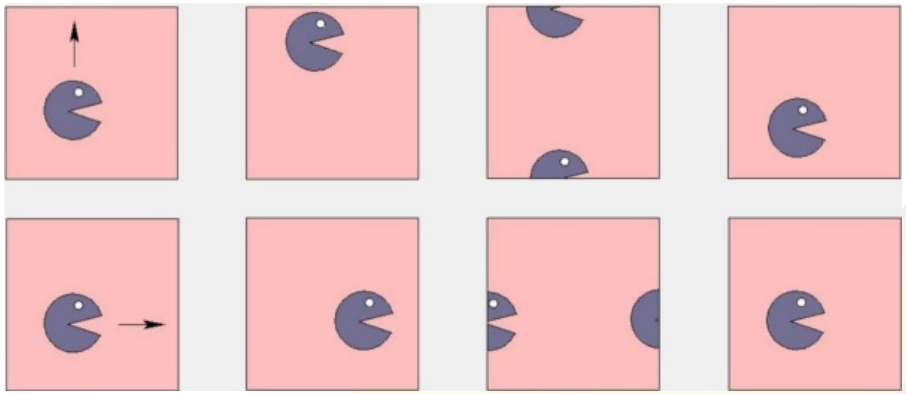

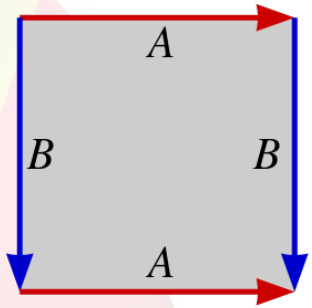

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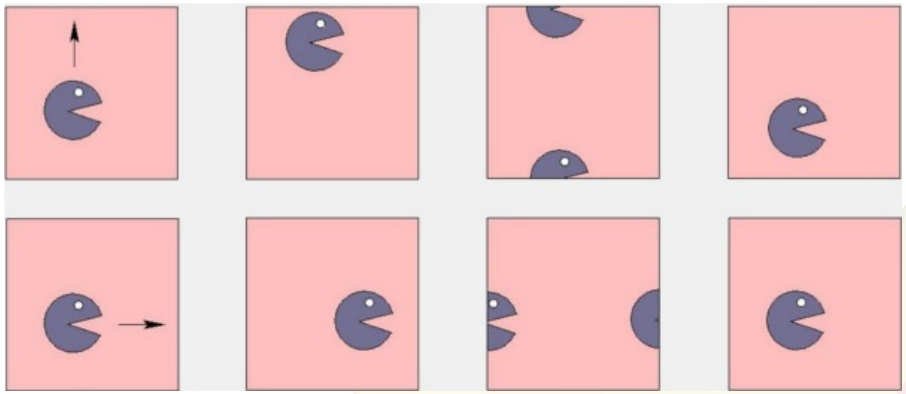

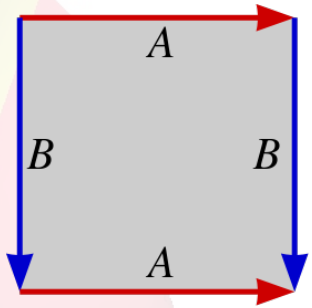

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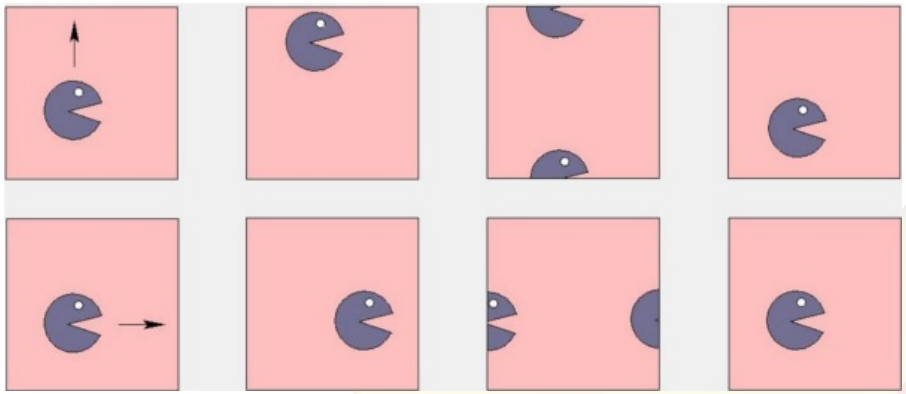

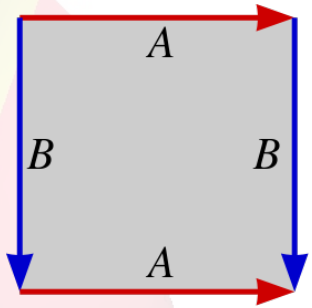

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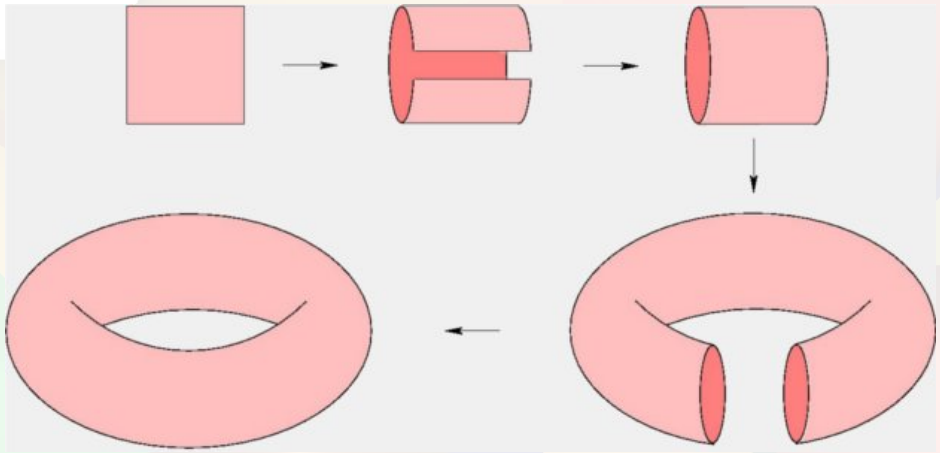

Pac man & Ms. Pac man live on a donut, also known as a torus.

Life on a donut

What is life like for pac man?

Life on a donut

What is life like for pac man?

- If you look around the donut, you will see another copy of yourself!

Let's play some torus games!

www.geometrygames.org

Shape of space: 3-D pac man

A 3-D version of pac man gives a 3D donut.

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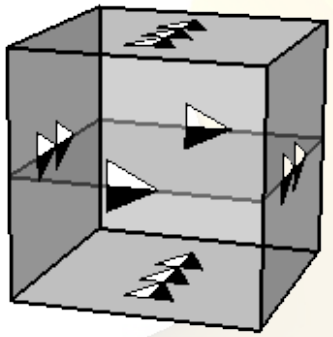
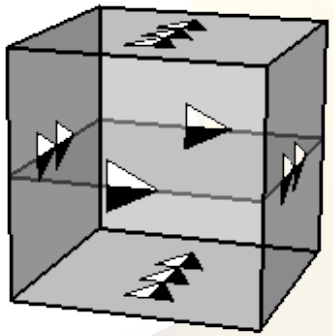
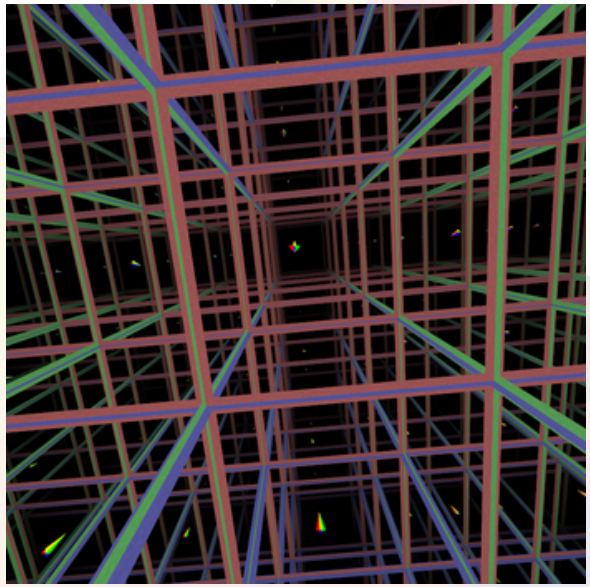


Shape of space: 3-D pac man

A 3-D version of pac man gives a 3D donut.





Let's play a 3D torus game!

Mathematical billiards

Some of Maryam Mirzakhani's mathematical research tells us about mathematical billiards.

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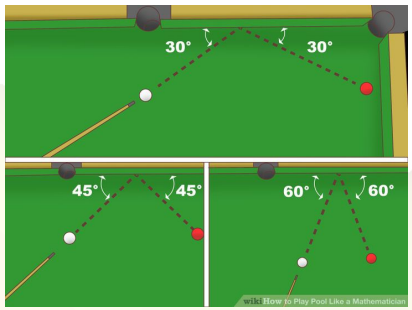
Idealised mathematical billiards

- No jumping, no spin, no friction
- Balls bounce off walls and continue indefinitely.

Mathematical billiards

We can ask many questions in mathematical billiards:

- In what ways can you bounce off the walls?



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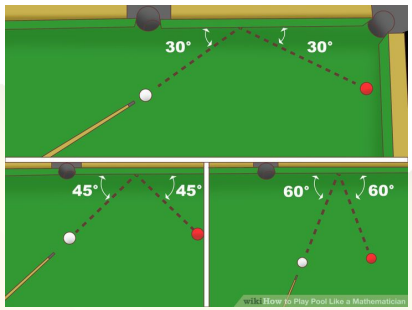

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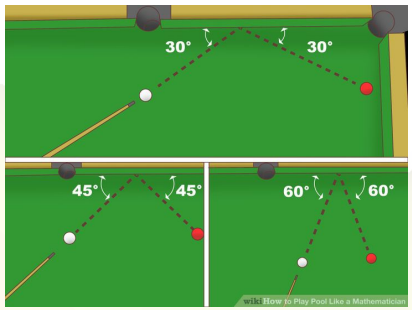

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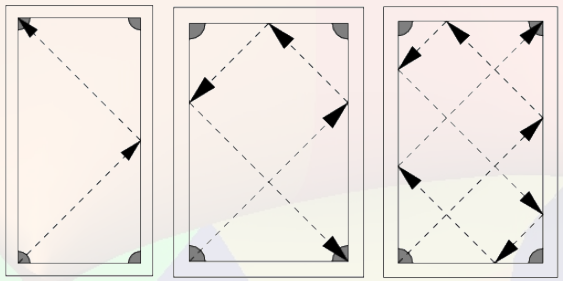
- Can you bounce 17 times and return to initial state?



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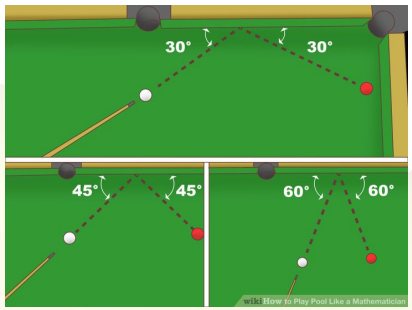

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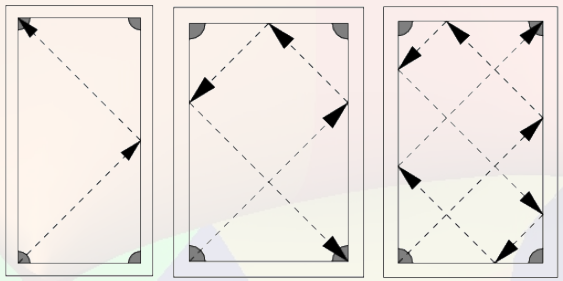
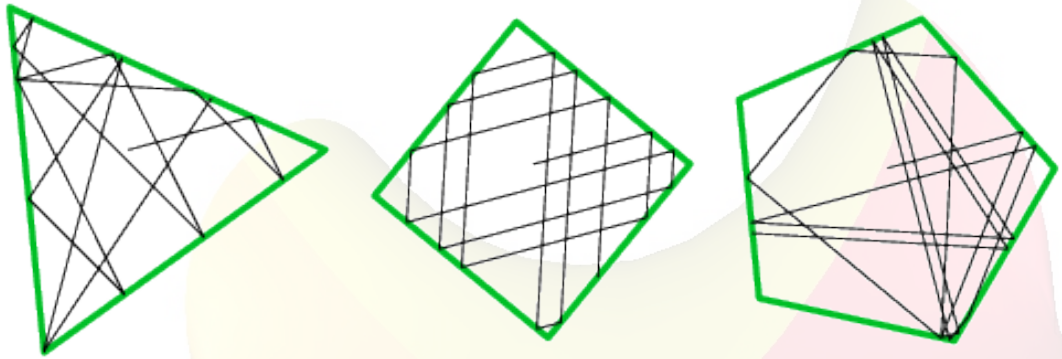


- Can the ball pass through every point on the table?

Mathematical billiards

- What if you change the *geometry* of the table — a more complicated shape?



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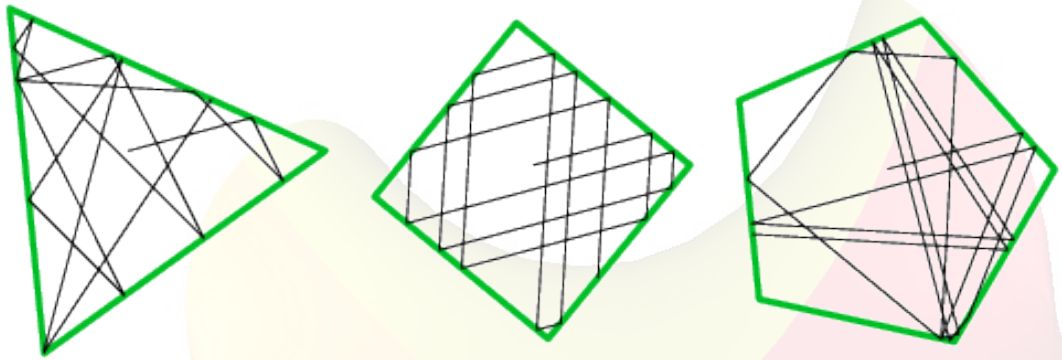

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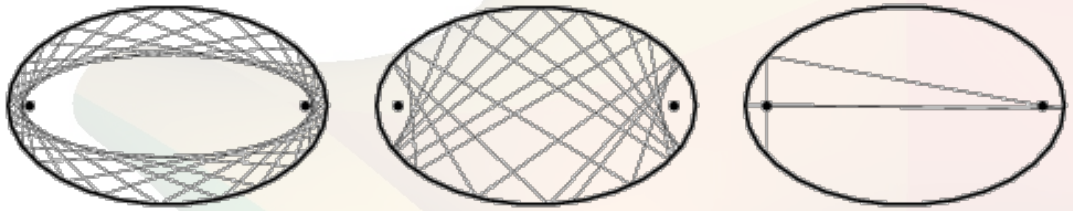

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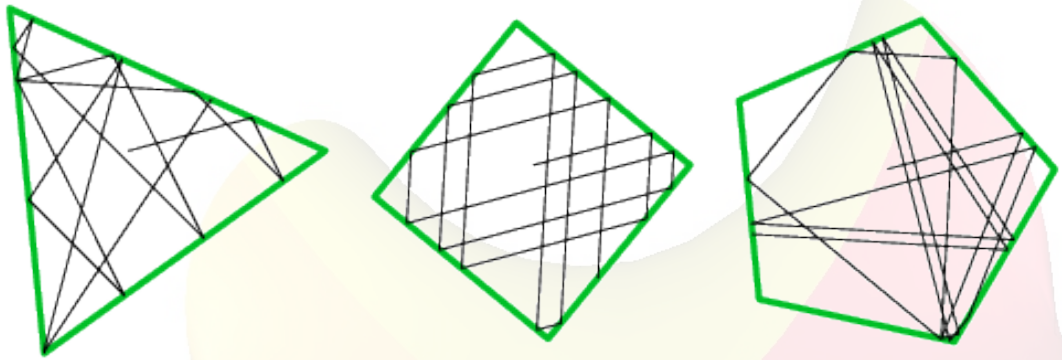
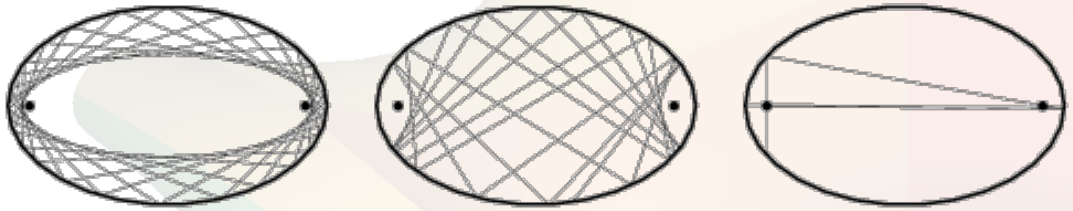
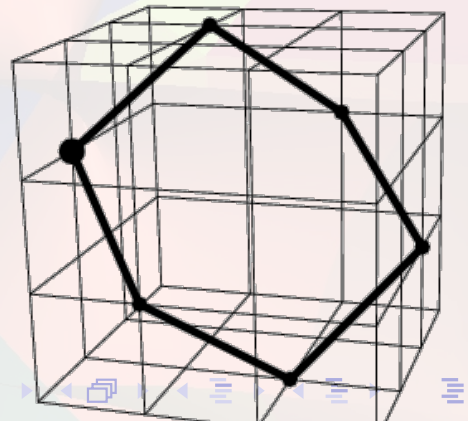


- What if you allow the table to be *curved*, or *3-dimensional*?





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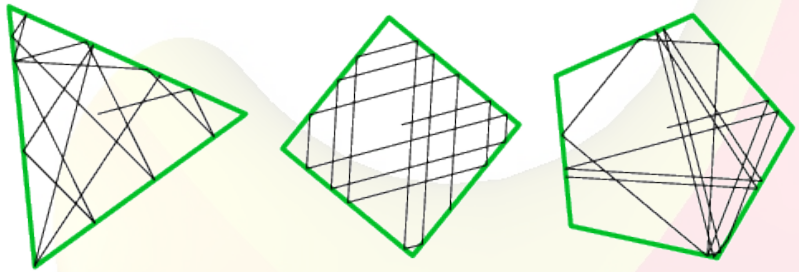

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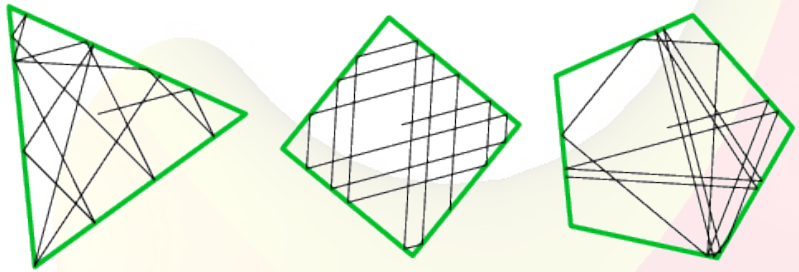

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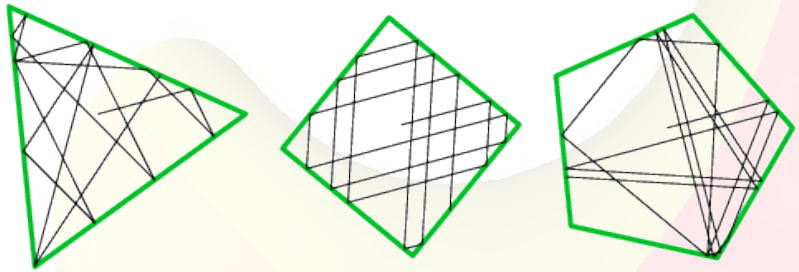

On some it is a little harder:

Why illumination?

- In a room with mirrored walls, if you light a candle at one point, does it illuminate every point in the room?

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It's not always possible to illuminate every point in the room.

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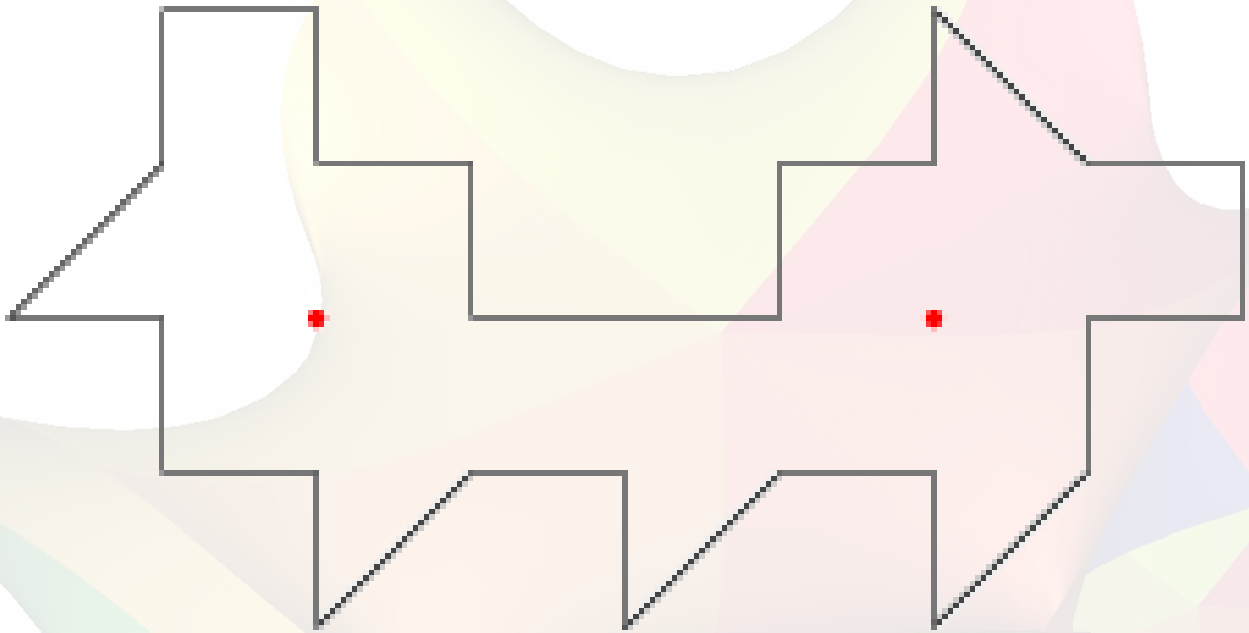

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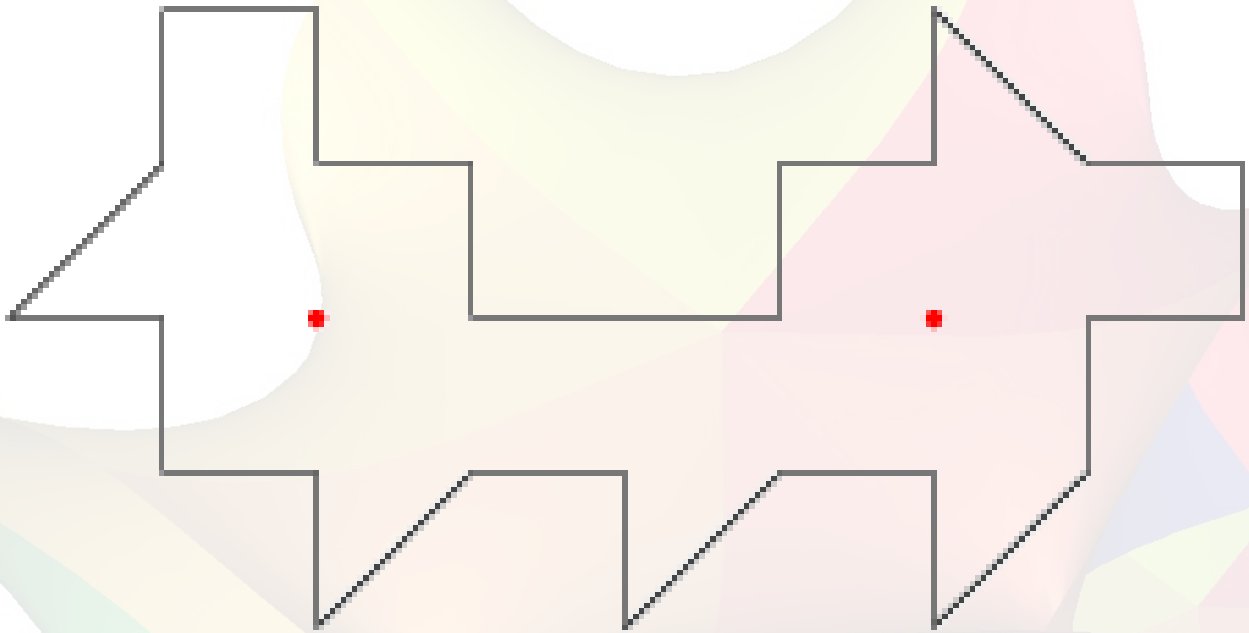

Even in Tokarsky's room, the candle comes close to lighting up the whole room!

An illumination theorem

- Can you design a room with straight mirrored walls, in which a candle can be placed, that leaves a whole region in the dark?

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- the candle lights up the entire room, except possibly it misses a finite number of points.

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- Be curious! Think critically!

Thanks for listening!

Daniel.Mathews@monash.edu

Get the games/software at

www.geometrygames.org

In memoriam

Maryam Mirzikhani 1977–2017

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